Mathematics Level 2

1. If $3^{x-3} + 10 = 19$, then $x =$

   (A) 3
   (B) 4
   (C) 5
   (D) 6
   (E) 7

2. A number's prime factors are 2, 5, 7, 13, and 31. Which of the following must be a factor of the number?

   (A) 4
   (B) 6
   (C) 10
   (D) 15
   (E) 25

3. What is the slope of the line that passes through the points (5, 4) and (–2, 3)?

   (A) 0.14
   (B) 0.20
   (C) 0.33
   (D) 5.00
   (E) 7.00

4. What is the magnitude of the vector $\mathbf{v} = (5, 0, 7)$?

   (A) 0.0
   (B) 3.5
   (C) 4.2
   (D) 8.6
   (E) 12.0

5. $\frac{a^{-b}}{a^{a}} \# a^{2}$ is equivalent to

(A) a^{-a-b+2}

(B) a^{a-b+2}

(C) a^{a+b-2}

(D) $a^{-\frac{2b}{a}}$

(E) $-\frac{2b}{a}$

6. If f is a linear function with nonzero slope, and $c < d$, which of the following must be FALSE?

(A) $f(c) = f(d)$

(B) $f(c) \neq f(d)$

(C) $f(c) > f(d)$

(D) $f(c) < f(d)$

(E) $f(c) = 0$ or $f(d) = 0$

7. If $\sin \theta = \frac{4}{5}$, then $\cos \theta =$

(A) $\frac{4}{5}$

(B) $\frac{5}{4}$

(C) $\frac{\pi}{2} - \frac{4}{5}$

(D) $\frac{\pi}{2} - \theta$

(E) $\frac{3}{5}$

8. For which of the following values of x is $f(x) = \frac{7}{(x^4 - 5x^3 - x^2 + 36x)}$ defined?

(A) –3

(B) 0

(C) 2

(D) 3

(E) 6

If $a + b = 0$, then $|a| = |b|$.

9. If a and b are real numbers, which of the following can be inferred from the statement above?

(A) If $|a| = |b|$, then $a + b = 0$.

(B) If $|a| \neq |b|$, then $a + b \neq 0$.

(C) If $a + b \neq 0$, then $|a| \neq |b|$.

(D) If $a - b = 0$, then $|a| \neq |b|$.

(E) If $|a| \neq |b|$, then $a - b = 0$.

10. The mean of 7 numbers is 15. When an 8th number is added, the mean decreases to 12. What is the 8th number?

(A) –12

(B) –9

(C) 0

(D) 8

(E) 12

11. If $f(x) = 3x$, $g(x) = 5x + 3$, and $h(x) = 1 - x^2$, then $f(g(h(x)))=$

(A) $15x^2 + 15$

(B) $-15x^2 + 18$

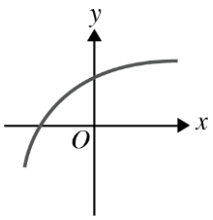
(C) $-15x^2 + 24$

(D) $-225x^2 + 90x - 8$

(E) $-225x^2 - 90x - 8$

12. Which of the following is NOT a function of x?

(A)



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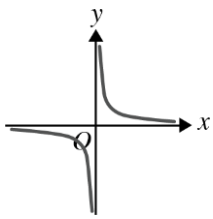

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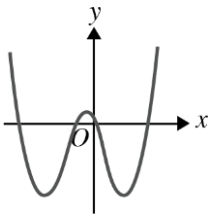
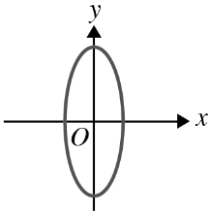
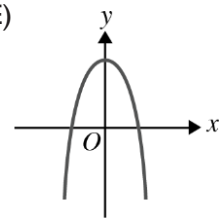


(D)



(E)



13. If a line is perpendicular to $y = \frac{1}{2}x - 3$, then its equation could be

(A) $y = -\frac{1}{2}x + 3$

(B) $y = 3x - \frac{1}{2}$

(C) $y = \frac{1}{\frac{1}{2}x - 3}$

(D) $y = 2x - 3$

(E) $y = -2x - 3$

14. If a pentagon P with vertices at $(-2, -4)$, $(-4, 1)$, $(-1, 4)$, $(2, 4)$, and $(3, 0)$ is reflected across the line $y = x$ to get a new pentagon, P', then one of the vertices of P' is

(A) $(0, -3)$

(B) $(4, 1)$

(C) $(2, 2)$

(D) $(4, -2)$

(E) $(-4, -2)$

15. Two poles stand on opposite sides of a level street that is 12 meters wide. If one pole is 5 meters taller than the other, what is the angle of elevation between the tops of the poles?

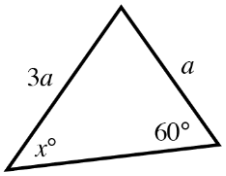
(A) $\sin^{-1}\left(\frac{5}{12}\right)$

(B) $\sin^{-1}\left(\frac{12}{13}\right)$

(C) $\tan^{-1}\left(\frac{5}{12}\right)$

(D) $\tan^{-1}\left(\frac{5}{13}\right)$

(E) $\cos^{-1}\left(\frac{5}{12}\right)$

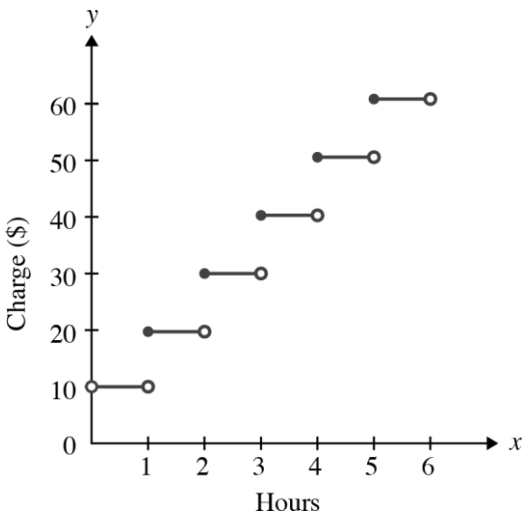


16. What is the value of $\sin(x°)$?

(A) 0.289
(B) 0.342
(C) 0.500
(D) 16.778
(E) Cannot be determined from the given information

17. Which of the following points is NOT a solution to both the inequalities $y > 9x - 8$ and $y < -x + 8$?

(A) $(-2, 15)$
(B) $(-1, 5)$
(C) $(0, 0)$
(D) $(1, 5)$
(E) $(1, 2)$



18. A lawn mowing company charges its customers according to the step function $y = 10\,x + 1$, for all $x > 0$, as shown in the graph above. If a customer's lawn takes 2 hours and 17 minutes to mow, how much does the company charge?

(A) \$32.83

(B) \$30.00

(C) \$22.83

(D) \$22.00

(E) \$20.00

19. What is the range of the function $y = 5 + 3\sin(\mathrm{r} - x)$?

(A) $-3 \le y \le 3$

(B) $-2 \le y \le 8$

(C) $0 \le y \le 6$

(D) $2 \le y \le 8$

(E) $y \in \mathrm{R}$

20. If $g(x) = f(-x)$ for all real numbers x, and if $(3, 2)$ is a point on the graph of g, which of the following points MUST be on the graph of f?

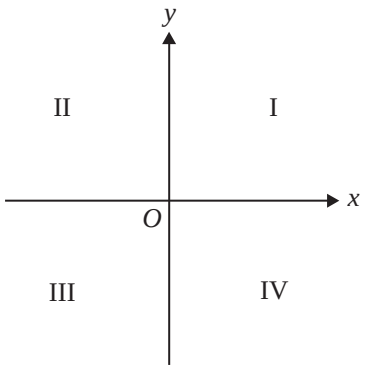
(A) $(3, 2)$

(B) $(3, -2)$

(C) $(-3, 2)$

(D) $(-3, -2)$

(E) $(2, 3)$



21. The figure above shows xy-coordinate space divided into four quadrants. If an angle measuring $a°$ is located in Quadrant II, then which of the following statements must be true?

I. $\cos(90° - a°)$ is positive
II. $\tan(a°)$ is positive
III. a is positive

(A) I only

(B) II only

(C) I and III

(D) II and III

(E) I, II, and III

22. If $f(x) = x^3 + 2x^2 - 9x - 18$, which of the following statements is true?

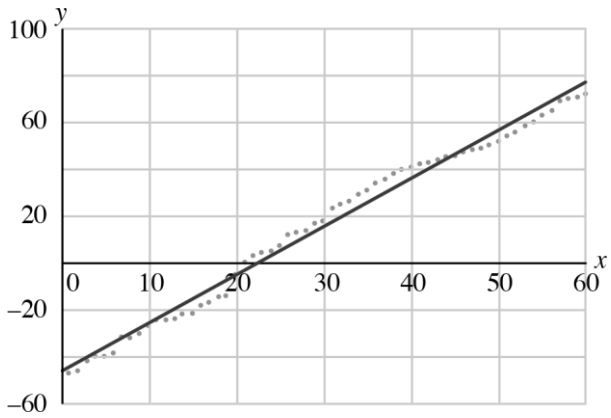
(A) $f(x) = 0$ has three real solutions.

(B) $f(x) \geq -18$ for all $x \geq 0$.

(C) $f(x) \leq -18$ for all $x \leq 0$.

(D) The function $f(x)$ is decreasing for $x \leq -3$.

(E) The function $f(x)$ is increasing for $x \geq -3$.



23. If a least-squares linear regression is used to model data, as shown in the graph above, what is a reasonable y–value estimate for an x–value of 197?

(A) 202

(B) 245

(C) 352

(D) 459

(E) 512

24. If $f:(x, y) \rightarrow (x + y, 2y - x)$ for every coordinate pair in the xy-plane, for what points (x, y) is it true that $f:(x, y) \rightarrow (x, y)$?

(A) The set of points (x, y) such that $x = 0$

(B) The set of points (x, y) such that $y = 0$

(C) The set of points (x, y) such that $x = y$

(D) The set of points (x, y) such that $x = 2y$

(E) $(0, 0)$ only

25. In Canada in 2014, the average wholesale price of soybeans was \$0.24 per pound. In 2015, the average wholesale price of soybeans was \$0.16 per pound. If a retailer purchased 20,000 pounds of soybeans in 2014 and in 2015, what was the percent change in the retailer's expenses from 2014 to 2015?

(A) –8%

(B) –33%

(C) –50%

(D) 8%

(E) 33%

26. If $f^{-1}(x) = \sqrt{7x^3}$ for $x \geq 0$, then $f(6) =$

(A) 0.6297

(B) 0.9499

(C) 1.7261

(D) 38.8844

(E) 136.0233

27. The cross-sectional area of a uranium nucleus is 1 *barn*, or 10^{-28} square meters. The amount of area required to sustain a cow for a year is 1 *cow's grass*, or 2.48×10^4 square meters. How many barns are in 1 cow's grass?

(A) 4.03×10^{-33}

(B) 4.03×10^{-28}

(C) 2.48×10^{-28}

(D) 2.48×10^{28}

(E) 2.48×10^{32}

$$A = \{-10, -5, -3, -1, 0, 1, 3, 5, 10\}$$

28. The elements of Set *A*, shown above, are multiplied by 2 to get Set *B*. Which of the following is true about Set *B*?

(A) The mean of *B* is greater than the mean of *A*.

(B) The median of *B* is greater than the median of *A*.

(C) The median of *B* is less than the median of *A*.

(D) The standard deviation of *B* is equal to the standard deviation of *A*.

(E) The standard deviation of *B* is exactly twice the standard deviation of *A*.

29. If $f(2x) = x + 5$ and $f(g(6)) = 13$, then $2g(6) =$

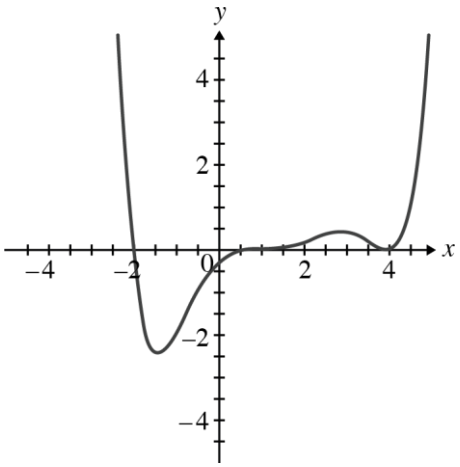
(A) 6

(B) 16

(C) 32

(D) 36

(E) 64



30. Which of the following functions could produce the graph above?

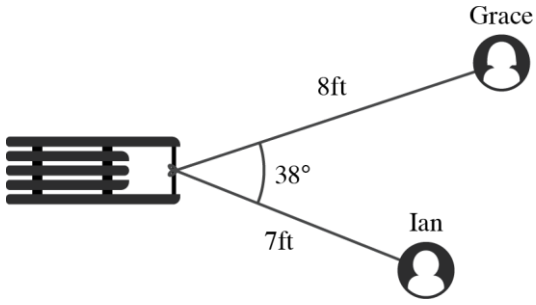
(A) $f(x) = 0.01(x - 1)(x - 4)(x + 2)$

(B) $f(x) = 0.01(x + 1)^3(x + 4)^2(x - 2)$

(C) $f(x) = 0.01(x + 1)^2(x + 4)^3(x - 2)^2$

(D) $f(x) = 0.01(x - 1)^3(x - 4)^2(x + 2)$

(E) $f(x) = 0.01(x - 1)^2(x - 4)^3(x + 2)^2$



31. Grace and Ian are working together to pull a sled, as shown in the figure above. If the angle between their ropes is 38°, what is the distance between them, to the nearest foot?

(A) 4

(B) 5

(C) 6

(D) 7

(E) 8

32. If $f(x) = 2x^3 + kx^2 - 2x - 3$ and $x - 1$ is a factor of $f(x)$, then $k =$

(A) –1
(B) 0
(C) 2
(D) 3
(E) 5

33. How many different possible committees of 5 people can be chosen from a group of 15 people?

(A) 75
(B) 120
(C) 225
(D) 3,003
(E) 3,628,800

34. If matrix A has dimensions 2×7 and matrix B has dimensions 7×5, what are the dimensions of the product matrix AB?

(A) 2×2
(B) 2×5
(C) 5×2
(D) 7×7
(E) The product AB does not exist.

35. For which of the following functions is the range equal to all real numbers?

(A) $f(x) = \frac{1}{2}x^3 - \frac{1}{5}x$

(B) $f(x) = x(3x^5 + 2x)$

(C) $f(x) = 112x^{14} - 23x^8 - 14x$

(D) $f(x) = \frac{2}{3}x^3(10x^3)(12x^3)$

(E) $f(x) = -3x^2 - x + \frac{5}{13}x^2$

36. $\frac{\log_3 1,000,000}{\log_3 1,000} =$

(A) 1,000

(B) 100

(C) 20

(D) 10

(E) 2

37. A positive integer n is called "powerful" if, for every prime factor p of n, p^2 is also a factor of n. An example of a powerful number is

(A) 240

(B) 297

(C) 300

(D) 336

(E) 392

38. The first three terms of a geometric sequence are $a_1 = 1$, $a_2 = -3$, and $a_3 = 9$. What is the formula for the n^{th} term in the sequence?

(A) $a_n = 3^n$

(B) $a_n = 3^{n-1}$

(C) $a_n = (-3)^n$

(D) $a_n = (-3)^{n-1}$

(E) $a_n = (-3)^{2n-1}$

39. $(i + 1)(5 - 5i)(5 + 5i) =$

(A) $50 + 50i$

(B) $50 - 50i$

(C) $25 + 25i$

(D) $25 - 25i$

(E) 0

40. If the distance between point R (a, a, a) and point J $(6, -2, 0)$ is 10, then the value of a could be

(A) $\frac{10}{3}$

(B) 4

(C) 5

(D) 6

(E) 10

41. Under which of the following conditions must $\frac{a+b}{a-b}$ be negative?

(A) $b = -a$

(B) $b < 0 < a$

(C) $a < 0 < b$

(D) $|b| < |a|$

(E) $|a| < |b|$

42. A circle passes through the points (3, 4) and (5, 7). Which of the following points CANNOT lie on the circle?

(A) (–2, –1)

(B) (3, 2)

(C) (5, 5)

(D) (6, 4)

(E) (–1, –2)

43. If $\cos(2x) = \frac{1}{2}$, what is a possible value for x?

(A) 420°

(B) 60°

(C) –45°

(D) –150°

(E) –720°

44. What expression can replace a in the equation $^{x}/_{64}\ h\ ^{\wedge}{:}^{y}/_{64}\ h = {}^{a}/_{64}$?

(A) $x + y$

(B) $x - y$

(C) $\dfrac{1}{x + y}$

(D) $\dfrac{1}{\frac{1}{x} + \frac{1}{y}}$

(E) $\dfrac{1}{x} + \dfrac{1}{y}$

45. At what value of x does the function $f(x) = x + 5 - \dfrac{x - 3}{x^2 - 1}$ intersect its oblique asymptote?

(A) -3

(B) 1

(C) 3

(D) 5

(E) $f(x)$ does not cross any of its asymptotes.

46. The shape of the graph represented by the equations $\begin{cases} x = \cos t \\ y = \sin t \end{cases}$, for $0 \le t \le \pi$, is

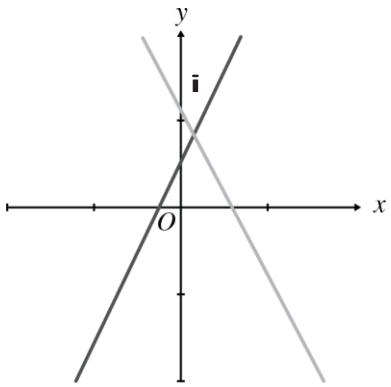
(A) a circle

(B) a semicircle

(C) a sigmoid

(D) a parabola

(E) a line



47. The graph above shows the lines $y = 2x + 7$ and $y = -2x + 12$. What is the measure of angle θ, in degrees?

(A) 30.00

(B) 36.87

(C) 45.00

(D) 53.13

(E) 126.9

48. A meteorologist reports that there is a 30% probability of rain and no sun. If there is a 40% probability of no rain, then the probability of both rain and sun is

(A) 0.16

(B) 0.24

(C) 0.30

(D) 0.50

(E) 0.60

49. Alex grows an initial culture of 100 *Rhizopus stolonifer* fungi on a sample of bread. She wants to model the growth of the fungi according to the exponential equation $A = Pe^{rt}$, where A is the final number of fungi, P is the initial number, r is the growth rate, and t is time elapsed in hours. If after 5 hours she measures the number of fungi to be 750, what is the value of r?

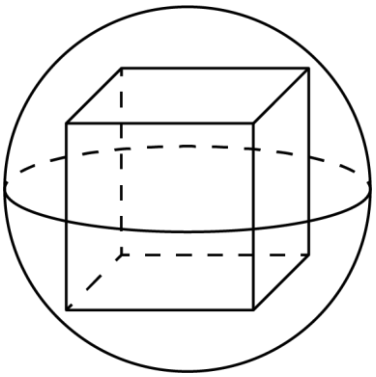
(A) 0.403

(B) 0.863

(C) 2.015

(D) 4.317

(E) 7.500



50. What is the surface area of a cube inscribed in a sphere with a radius of 8, as shown above?

(A) 85.3

(B) 512.0

(C) 768.0

(D) 788.3

(E) 804.3